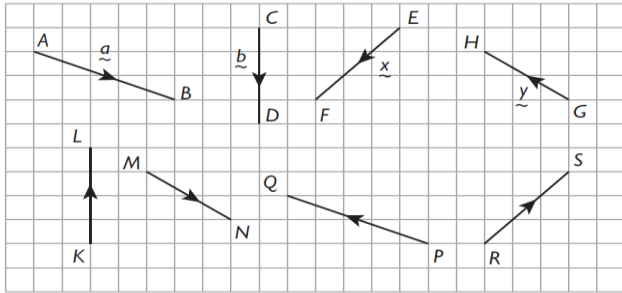


# VEKTOR

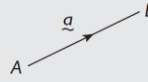
**Nyatakan vektor yang ditunjukkan dalam rajah berikut.**  
State the vectors shown in the following diagram.



1.3

## FAKTA UTAMA

Tatatanda vektor:  
Vector notation:



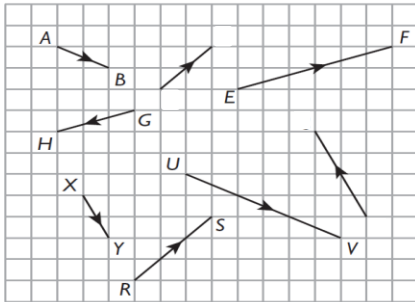
$$\vec{AB} = \underline{a}$$

$$\vec{BA} = -\vec{AB} = -\underline{a}$$

Magnitud  $\vec{AB}$  :  $|\vec{AB}| = |\underline{a}|$   
Magnitude of  $\vec{AB}$

**Tentukan pasangan vektor yang selari dan nyatakan hubungannya.**  
Determine the vectors that are parallel and state their relationship.

1.6



## FAKTA UTAMA

Dua vektor,  $\vec{PQ}$  dan  $\vec{RS}$ , adalah selari jika dan hanya jika  $\vec{PQ} = k\vec{RS}$ , dengan keadaan  $k$  ialah pemalar.

Two vectors,  $\vec{PQ}$  and  $\vec{RS}$ , are parallel if and only if  $\vec{PQ} = k\vec{RS}$ , where  $k$  is a constant.

### CONTOH #1

$$(m + 2n - 5)\underline{x} - (m - n + 4)\underline{y} = \underline{0}$$

$$(m + 2n - 5)\underline{x} = (m - n + 4)\underline{y}$$

$$m + 2n - 5 = 0 \quad \dots\dots \textcircled{1}$$

$$m - n + 4 = 0 \quad \dots\dots \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}: \quad 3n - 9 = 0$$

$$3n = 9$$

$$n = 3$$

Gantikan  $n = 3$  ke dalam  $\textcircled{2}$ .

$$m - 3 + 4 = 0$$

$$m = -1$$

#### FAKTA UTAMA

Diberi dua vektor,  $\underline{a}$  dan  $\underline{b}$ , adalah bukan sifar dan tidak selari. Jika  $m\underline{a} = n\underline{b}$ , maka  $m = n = 0$ .

Given two vectors,  $\underline{a}$  and  $\underline{b}$ , are non-zero and non-parallel. If  $m\underline{a} = n\underline{b}$ , then  $m = n = 0$ .

### CONTOH #2

Diberi  $\vec{PQ} = 12\underline{x}$  dan  $\vec{QR} = 16\underline{x}$ , tunjukkan bahawa P, Q dan R adalah segaris.

Given  $\vec{PQ} = 12\underline{x}$  and  $\vec{QR} = 16\underline{x}$ , show that P, Q and R are collinear.

$$\vec{PQ} = 12\underline{x}$$

$$= \frac{3}{4}(16\underline{x})$$

$$\vec{PQ} = \frac{3}{4}\vec{QR}$$

PQ dan QR adalah selari dan titik Q ialah titik sepunya. Maka, P, Q dan R adalah segaris.

### #3

#### CONTOH

(a)  $\underline{a} + 2\underline{a} + \frac{1}{3}\underline{a}$

$$= \left(1 + 2 + \frac{1}{3}\right)\underline{a}$$

$$= 3\frac{1}{3}\underline{a}$$

(b)  $(2\underline{a} + \underline{b}) + \left(3\underline{a} + \frac{1}{2}\underline{b}\right)$

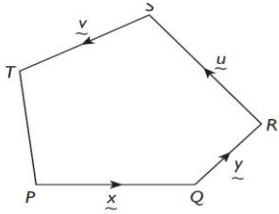
$$= (2 + 3)\underline{a} + \left(1 + \frac{1}{2}\right)\underline{b}$$

$$= 5\underline{a} + 1\frac{1}{2}\underline{b}$$

1.  $\underline{a} + \frac{1}{2}\underline{a} + \frac{1}{3}\underline{a}$

2.  $4\underline{x} + \frac{1}{3}\underline{x} + \frac{1}{4}\underline{x}$

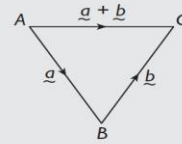
**CONTOH #4**



- (a)  $\underline{x} + \underline{y} = \overrightarrow{PR}$
- (b)  $\underline{u} + \underline{v} = \overrightarrow{RT}$
- (c)  $\overrightarrow{QR} + \overrightarrow{RS} = \overrightarrow{QS}$
- (d)  $\overrightarrow{PQ} + \overrightarrow{QT} = \overrightarrow{PT}$

**FAKTA UTAMA**

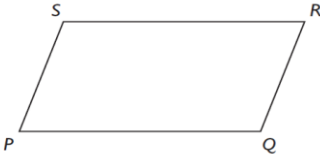
Hukum segi tiga:  
Triangle law:



$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

**CONTOH #5**

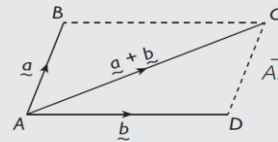
PQRS ialah segi empat selari.  
PQRS is a parallelogram.



- (a)  $\overrightarrow{PQ} + \overrightarrow{PS} = \overrightarrow{PR}$
- (b)  $\overrightarrow{QR} + \overrightarrow{QP} = \overrightarrow{QS}$
- (c)  $\overrightarrow{SR} + \overrightarrow{SP} = \overrightarrow{SQ}$
- (d)  $\overrightarrow{RS} + \overrightarrow{RQ} = \overrightarrow{RP}$

**FAKTA UTAMA**

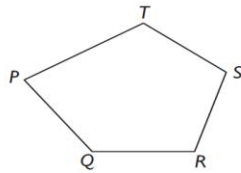
Hukum segi empat selari:  
Parallelogram law:



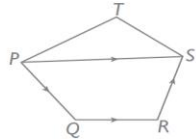
$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$$

**CONTOH #6**

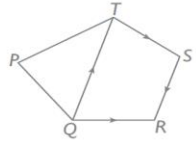
PQRST ialah pentagon.  
PQRST is a pentagon.



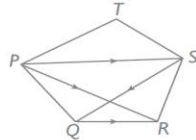
(a)  $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS} = \overrightarrow{PS}$



(b)  $\overrightarrow{QT} + \overrightarrow{TS} + \overrightarrow{SR} = \overrightarrow{QR}$

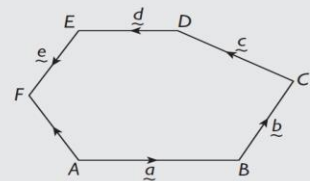


(c)  $\overrightarrow{PS} + \overrightarrow{SQ} + \overrightarrow{QR} = \overrightarrow{PR}$



**FAKTA UTAMA**

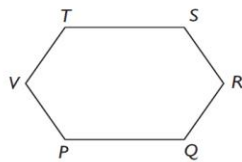
Hukum poligon:  
Polygon law:



$$\begin{aligned} \overrightarrow{AF} &= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} \\ &= \underline{a} + \underline{b} + \underline{c} + \underline{d} + \underline{e} \end{aligned}$$

**CONTOH #7**

PQRSTV ialah heksagon.  
PQRSTV is a hexagon.

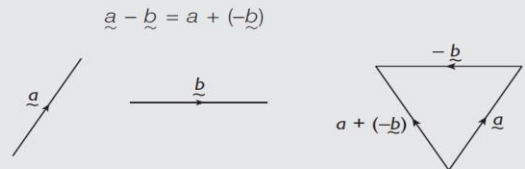


(a)  $\begin{aligned} \overrightarrow{PQ} - \overrightarrow{RQ} &= \overrightarrow{PQ} + (-\overrightarrow{RQ}) \\ &= \overrightarrow{PQ} + \overrightarrow{QR} \\ &= \overrightarrow{PR} \end{aligned}$

(b)  $\begin{aligned} \overrightarrow{TS} - \overrightarrow{QS} - \overrightarrow{VQ} &= \overrightarrow{TS} + \overrightarrow{SQ} + \overrightarrow{QV} \\ &= \overrightarrow{TV} \end{aligned}$

**FAKTA UTAMA**

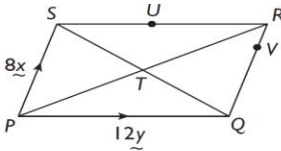
- Penolakan yang melibatkan dua vektor yang selari:  
Subtraction involving two parallel vectors:  
 $8x - 3x = 8x + (-3x) = 5x$
- Penolakan yang melibatkan dua vektor yang tidak selari:  
Subtraction involving two non-parallel vectors:



$$\underline{a} - \underline{b} = \underline{a} + (-\underline{b})$$

Ungkapkan setiap vektor dalam sebutan  $\underline{x}$  dan  $\underline{y}$ .  
Express each vector in terms of  $\underline{x}$  and  $\underline{y}$ .

**CONTOH #8**



PQRS ialah segi empat selari. Pepenjuru PR dan QS menyilang di T. U ialah titik tengah RS dan V terletak pada QR dengan keadaan QV : VR = 3 : 1.  
PQRS is a parallelogram. The diagonals PR and QS intersect at T. U is the midpoint of RS and V lies on QR such that QV : VR = 3 : 1.

$$\begin{aligned} \text{(a) } \vec{PU} &= \vec{PS} + \vec{SU} \\ &= \vec{PS} + \frac{1}{2} \vec{SR} \\ &= \vec{PS} + \frac{1}{2} \vec{PQ} \\ &= 8\underline{x} + \frac{1}{2}(12\underline{y}) \\ &= 8\underline{x} + 6\underline{y} \end{aligned}$$

$$\begin{aligned} \text{(b) } \vec{PV} &= \vec{PQ} + \vec{QV} \\ &= \vec{PQ} + \frac{3}{4} \vec{QR} \\ &= 12\underline{y} + \frac{3}{4}(8\underline{x}) \\ &= 6\underline{x} + 12\underline{y} \end{aligned}$$

$$\begin{aligned} \text{(c) } \vec{UV} &= \vec{UP} + \vec{PV} \\ &= -\vec{PU} + \vec{PV} \\ &= -8\underline{x} - 6\underline{y} + 6\underline{x} + 12\underline{y} \\ &= -2\underline{x} + 6\underline{y} \end{aligned}$$

**CONTOH #9**

Diberi O, A, B dan C ialah empat titik dengan keadaan  $\vec{OA} = 4\underline{a} - 2\underline{b}$ ,  $\vec{OB} = k\underline{a} + 3\underline{b}$  dan  $\vec{OC} = 5\underline{a} + 6\underline{b}$ .  
Given O, A, B and C are four points such that  $\vec{OA} = 4\underline{a} - 2\underline{b}$ ,  $\vec{OB} = k\underline{a} + 3\underline{b}$  and  $\vec{OC} = 5\underline{a} + 6\underline{b}$ .

(a) Ungkapkan  $\vec{AB}$  dan  $\vec{AC}$  dalam sebutan  $\underline{a}$  dan  $\underline{b}$ .  
Express  $\vec{AB}$  and  $\vec{AC}$  in terms of  $\underline{a}$  and  $\underline{b}$ .

$$\begin{aligned} \text{(a) } \vec{AB} &= \vec{OB} - \vec{OA} \\ &= (k\underline{a} + 3\underline{b}) - (4\underline{a} - 2\underline{b}) \\ &= (k - 4)\underline{a} + 5\underline{b} \\ \vec{AC} &= \vec{OC} - \vec{OA} \\ &= (5\underline{a} + 6\underline{b}) - (4\underline{a} - 2\underline{b}) \\ &= \underline{a} + 8\underline{b} \end{aligned}$$

(b) Jika titik A, B dan C adalah segaris, cari nilai k.  
If points A, B and C are collinear, find the value of k.

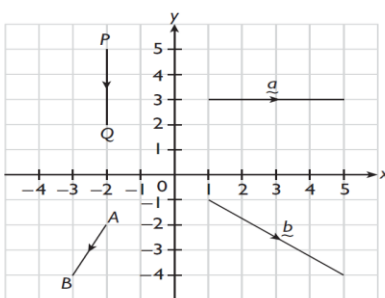
(b) A, B dan C adalah segaris.  
 $\vec{AB} = m\vec{AC}$ , dengan keadaan m ialah pemalar.  
 $(k - 4)\underline{a} + 5\underline{b} = m(\underline{a} + 8\underline{b})$   
 $k - 4 = m \dots\dots \textcircled{1}$   
 $5 = 8m$   
 $m = \frac{5}{8} \dots\dots \textcircled{2}$

Gantikan  $\textcircled{2}$  ke dalam  $\textcircled{1}$ .  $k - 4 = \frac{5}{8}$   
 $k = 4\frac{5}{8}$

**4.3 Vektor dalam Satah Cartes**

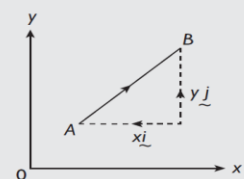
Ungkapkan setiap vektor dalam bentuk  $\begin{pmatrix} x \\ y \end{pmatrix}$  dan  $x\underline{i} + y\underline{j}$ . Seterusnya, cari magnitud bagi setiap vektor.  
Express each vector in the form  $\begin{pmatrix} x \\ y \end{pmatrix}$  and  $x\underline{i} + y\underline{j}$ . Hence, find the magnitude of each vector.

**CONTOH #10**



$$\begin{aligned} \text{(a) } \underline{a} &= \begin{pmatrix} 4 \\ 0 \end{pmatrix} = 4\underline{i} & \text{(b) } \underline{b} &= \begin{pmatrix} 4 \\ -3 \end{pmatrix} = 4\underline{i} - 3\underline{j} \\ |\underline{a}| &= 4 \text{ unit} & |\underline{b}| &= \sqrt{4^2 + (-3)^2} \\ & & &= 5 \text{ unit} \\ \text{(c) } \vec{PQ} &= \begin{pmatrix} 0 \\ -3 \end{pmatrix} = -3\underline{j} \\ |\vec{PQ}| &= 3 \text{ unit} \\ \text{(d) } \vec{AB} &= \begin{pmatrix} -1 \\ -2 \end{pmatrix} = -\underline{i} - 2\underline{j} \\ |\vec{AB}| &= \sqrt{(-1)^2 + (-2)^2} = \sqrt{5} \text{ unit} \end{aligned}$$

**FAKTA UTAMA**



- $\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix} = x\underline{i} + y\underline{j}$
- Magnitud  $\vec{AB}$ :  
Magnitude of  $\vec{AB}$ :  
 $|\vec{AB}| = \sqrt{x^2 + y^2}$

Tentukan vektor unit dalam arah setiap vektor berikut.  
Determine the unit vector in the direction of each of the following vectors.

**CONTOH #11**

$\underline{a} = 2\underline{i} - 3\underline{j}$

$|\underline{a}| = \sqrt{2^2 + (-3)^2}$   
 $= \sqrt{13} \text{ unit}$

$\hat{\underline{a}} = \frac{1}{\sqrt{13}}(2\underline{i} - 3\underline{j})$   
 $= \frac{2}{\sqrt{13}}\underline{i} - \frac{3}{\sqrt{13}}\underline{j}$

**FAKTA UTAMA**

- Jika  $\underline{a} = x\underline{i} + y\underline{j} = \begin{pmatrix} x \\ y \end{pmatrix}$ , maka  $|\underline{a}| = \sqrt{x^2 + y^2}$ .  
If  $\underline{a} = x\underline{i} + y\underline{j} = \begin{pmatrix} x \\ y \end{pmatrix}$ , then  $|\underline{a}| = \sqrt{x^2 + y^2}$ .
- $\hat{\underline{a}} = \frac{\underline{a}}{|\underline{a}|} = \frac{x\underline{i} + y\underline{j}}{\sqrt{x^2 + y^2}} = \frac{1}{\sqrt{x^2 + y^2}} \begin{pmatrix} x \\ y \end{pmatrix}$

**CONTOH #12**

$$\begin{aligned} 2\underline{u} + \underline{v} - 3\underline{w} &= 2(3\underline{i} + 5\underline{j}) + (2\underline{i} - \underline{j}) - 3(-4\underline{i} + 2\underline{j}) \\ &= 6\underline{i} + 10\underline{j} + 2\underline{i} - \underline{j} + 12\underline{i} - 6\underline{j} \\ &= 20\underline{i} - 3\underline{j} \end{aligned}$$

**CONTOH #13**

$$\begin{aligned} 3\underline{p} - \underline{q} + 4\underline{r} &= 3\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} + 4\begin{pmatrix} 5 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 9 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 20 \\ -4 \end{pmatrix} \\ &= \begin{pmatrix} 30 \\ -2 \end{pmatrix} \end{aligned}$$

**CONTOH #14**

Diberi  $\vec{OP} = 4\underline{i} - 8\underline{j}$ ,  $\vec{OQ} = 5\underline{i} + 2\underline{j}$  dan  $\vec{OP} - 2\vec{OQ} + 3\vec{OR} = \underline{0}$ , cari

Given  $\vec{OP} = 4\underline{i} - 8\underline{j}$ ,  $\vec{OQ} = 5\underline{i} + 2\underline{j}$  and  $\vec{OP} - 2\vec{OQ} + 3\vec{OR} = \underline{0}$ , find

- (a)  $\vec{OR}$ . (b)  $|\vec{OR}|$ . (c) vektor unit dalam arah  $\vec{OR}$ .  
the unit vector in the direction of  $\vec{OR}$ .

(a)  $\vec{OP} - 2\vec{OQ} + 3\vec{OR} = \underline{0}$

$$\begin{aligned} 3\vec{OR} &= 2\vec{OQ} - \vec{OP} \\ &= 2(5\underline{i} + 2\underline{j}) - (4\underline{i} - 8\underline{j}) \\ &= 10\underline{i} + 4\underline{j} - 4\underline{i} + 8\underline{j} \\ &= 6\underline{i} + 12\underline{j} \\ \vec{OR} &= 2\underline{i} + 4\underline{j} \end{aligned}$$

(b)  $|\vec{OR}| = \sqrt{2^2 + 4^2}$   
 $= \sqrt{20}$   
 $= 2\sqrt{5}$  unit

(c) Vektor unit dalam arah  $\vec{OR} = \frac{1}{2\sqrt{5}}(2\underline{i} + 4\underline{j})$   
 $= \frac{1}{\sqrt{5}}\underline{i} + \frac{2}{\sqrt{5}}\underline{j}$

1. Diberi  $\vec{OA} = 4\underline{i} - \underline{j}$ ,  $\vec{OB} = -2\underline{i} + 3\underline{j}$  dan  $\vec{OA} + 3\vec{OB} + 2\vec{OC} = \underline{0}$ , cari

Given  $\vec{OA} = 4\underline{i} - \underline{j}$ ,  $\vec{OB} = -2\underline{i} + 3\underline{j}$  and  $\vec{OA} + 3\vec{OB} + 2\vec{OC} = \underline{0}$ , find

- (a)  $\vec{OC}$ . (b)  $|\vec{OC}|$ . (c) vektor unit dalam arah  $\vec{OC}$ .  
the unit vector in the direction of  $\vec{OC}$ .

2. Koordinat titik A dan titik B masing-masing ialah (2, 3) dan (5, -6), dan O ialah asalan. P ialah satu titik pada AB dengan keadaan  $AP : PB = 2 : 1$ . Cari

The coordinates of points A and B are (2, 3) and (5, -6) respectively and O is the origin. P is a point on AB such that  $AP : PB = 2 : 1$ . Find

- (a)  $\vec{AB}$ . (b)  $\vec{OP}$ . (c) vektor unit dalam arah  $\vec{OP}$ .  
the unit vector in the direction of  $\vec{OP}$ .

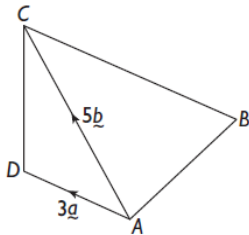
Jawab semua soalan.  
Answer all the questions.

1.  $\underline{a}$  dan  $\underline{b}$  ialah vektor bukan sifar dan tidak selari. Diberi  $(m - 5)\underline{a} = (3n + 4)\underline{b}$ , dengan keadaan  $m$  dan  $n$  ialah pemalar. Cari nilai  $m$  dan nilai  $n$ .  
 *$\underline{a}$  and  $\underline{b}$  are non-zero and non-parallel vectors. Given  $(m - 5)\underline{a} = (3n + 4)\underline{b}$ , where  $m$  and  $n$  are constants. Find the values of  $m$  and  $n$ .*

[2]

2. Rajah di bawah menunjukkan trapezium ABCD dengan keadaan  $\vec{BC} = 2\vec{AD}$ .

The diagram shows a trapezium ABCD such that  $\vec{BC} = 2\vec{AD}$ .



Ungkapkan dalam sebutan  $\underline{a}$  dan/atau  $\underline{b}$ :  
Express in terms of  $\underline{a}$  and/or  $\underline{b}$ :

- (a)  $\vec{DC} - \vec{AC}$                       (b)  $\vec{BA}$

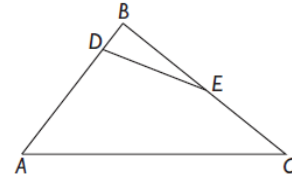
[3]

3. Rajah di bawah menunjukkan segi tiga ABC dengan  $\vec{AC} = 2\underline{a}$  dan  $\vec{AB} = 8\underline{b}$ .



4.2

The diagram shows a triangle ABC with  $\vec{AC} = 2\underline{a}$  and  $\vec{AB} = 8\underline{b}$ .



- (a) Cari  $\vec{BC}$  dalam sebutan  $\underline{a}$  dan  $\underline{b}$ .  
Find  $\vec{BC}$  in terms of  $\underline{a}$  and  $\underline{b}$ .
- (b) Diberi  $AB = 4DB$  dan  $BE = EC$ , cari  $\vec{DE}$  dalam sebutan  $\underline{a}$  dan  $\underline{b}$ .  
Given  $AB = 4DB$  and  $BE = EC$ , find  $\vec{DE}$  in terms of  $\underline{a}$  and  $\underline{b}$ .

[4]

4. Diberi bahawa vektor  $\underline{p} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$  dan vektor  $\underline{q} = \begin{pmatrix} k \\ 10 \end{pmatrix}$ , dengan keadaan  $k$  ialah pemalar.

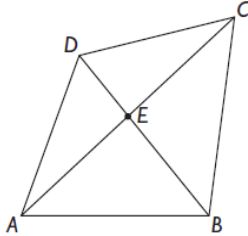
4.3

It is given that the vectors  $\underline{p} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$  and  $\underline{q} = \begin{pmatrix} k \\ 10 \end{pmatrix}$ , where  $k$  is a constant.

- (a) Ungkapkan vektor  $\underline{p} + \underline{q}$  dalam sebutan  $k$ .  
Express the vector  $\underline{p} + \underline{q}$  in terms of  $k$ .
- (b) Diberi  $|\underline{p} + \underline{q}| = 25$  unit, cari nilai positif  $k$ .  
Given  $|\underline{p} + \underline{q}| = 25$  units, find the positive value of  $k$ . [3]

Jawab semua soalan.  
Answer all the questions.

- 1.** Rajah di bawah menunjukkan sisi empat  $ABCD$ . Garis lurus  $AC$  bersilang dengan garis lurus  $BD$  di titik  $E$ .  
The diagram shows a quadrilateral  $ABCD$ . The straight line  $AC$  intersects the straight line  $BD$  at point  $E$ .



Diberi:  
Given:

$$BE : ED = 3 : 2, \vec{AB} = 20\vec{x}, \vec{AD} = 15\vec{y},$$

$$\vec{BC} = -4\vec{x} + 18\vec{y}$$

- (a) Ungkapkan dalam sebutan  $\vec{x}$  dan  $\vec{y}$ :  
Express in terms of  $\vec{x}$  and  $\vec{y}$ :

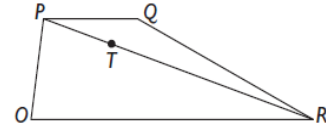
(i)  $\vec{BD}$                       (ii)  $\vec{AE}$

[3]

- (b) Cari nisbah  $AE : EC$ .  
Find the ratio  $AE : EC$ .

[5]

- 2.** Rajah di bawah menunjukkan trapezium  $OPQR$  dan titik  $T$  terletak pada  $PR$ .  
The diagram shows a trapezium  $OPQR$  and point  $T$  lies on  $PR$ .



- (a) Diberi bahawa  $\vec{OP} = 4\vec{a}$ ,  $\vec{OR} = 24\vec{b}$  dan  $\vec{OR} = 3\vec{PQ}$ . Ungkapkan dalam sebutan  $\vec{a}$  dan  $\vec{b}$ :

It is given that  $\vec{OP} = 4\vec{a}$ ,  $\vec{OR} = 24\vec{b}$  and  $\vec{OR} = 3\vec{PQ}$ . Express in terms of  $\vec{a}$  and  $\vec{b}$ :

(i)  $\vec{PR}$                       (ii)  $\vec{OQ}$

[3]

- (b) Diberi bahawa  $\vec{PT} = m\vec{PR}$ , dengan keadaan  $m$  ialah pemalar. Cari nilai  $m$  jika titik-titik  $O$ ,  $T$  dan  $Q$  adalah segaris.

It is given that  $\vec{PT} = m\vec{PR}$ , such that  $m$  is a constant. Find the value of  $m$  if the points  $O$ ,  $T$  and  $Q$  are collinear.

[5]